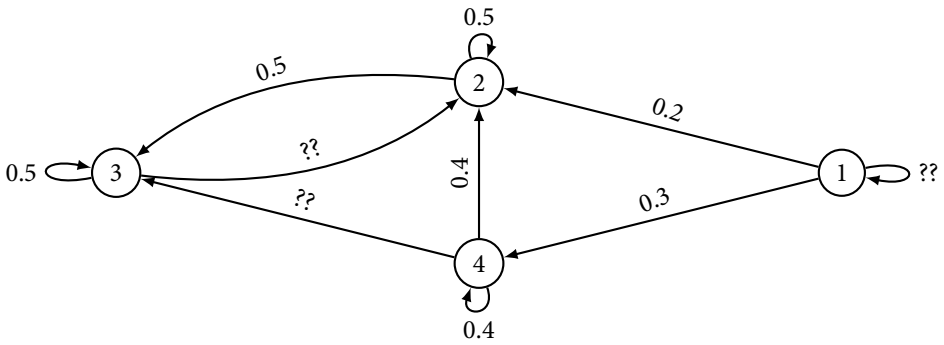
Review Problems for Exam 2

Problem 1. Consider the following transition-probability diagram for a Markov chain with four states – note that a few of the arc labels have missing values (??). Write the corresponding one-step transition matrix. Your matrix must not have any missing values.



Problem 2. An autonomous UUV has been programmed to move randomly between five regions according to a Markov chain. Looking at the documentation written by the programmer, you find the following one-step transition matrix:

$$\mathbf{P} = \begin{bmatrix} 0.40 & 0.10 & 0.25 & 0 & 0.25 \\ 0.20 & 0.30 & 0 & 0.25 & 0.25 \\ 0 & 0 & 0.75 & 0.25 & 0 \\ 0 & 0 & 0.50 & 0.50 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

a. Suppose that the UUV starts in each of the five regions with equal probability. What is the probability that the UUV is in region 4 after 3 movements?

b. Do regions 2 and 3 form an irreducible set of states? Why or why not?

c. Suppose the UUV reaches region 4. What is the long-run fraction of time it spends in region 4?

d. What is the probability that the UUV starts in region 1 and eventually ends up in region 5?

Problem 3. The statistician of the Simplexville Stars basketball team believes that players experience "hot hand" and "cold hand". According to her data, a player who made her last two shots has a 35% chance of making her next shot. On the other hand, a player who missed her last two shots has a 25% chance of making her next shot. Otherwise, a player has a 30% of making her next shot.

Model a player's shooting as a Markov chain by defining:

- the state space and the meaning of each state in the setting's context,
- the meaning of one time step in the setting's context, and
- the one-step transition probabilities.

Problem 4. Consider a model of officer promotion that defines the state of the system to be the rank of the officer (including separation and retirement) and the time index to be the number of years of service. Describe what assumptions need to be made in order for the Markov property and the time-stationarity property to hold. (You do not need to discuss whether these assumptions are realistic.)